A SELF-TEACHING GUIDE

All the Math You'll Ever Need

3RD EDITION

Carolyn C. WHEATER
Steve SLAVIN

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A Wiley Imprint
111 River St, Hoboken, NJ 07030
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Library of Congress Cataloging-in-Publication Data is Available:

ISBN 9781119719182 (paperback)
ISBN 9781119719229 (epdf)
ISBN 9781119719199 (ebook)

Cover Design: Paul McCarthy
Cover Image: © Frankramspott/Getty Images

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How to Use This Book

This book is organized by chapter with periodic self-tests throughout each chapter. Their purpose is to make sure you comprehend material before moving on. If you find that you have made an error, look back at the preceding material to make sure you understand the correct answer. The information is arranged so that it builds on what comes before. To fully understand the information at the end of a chapter, you must first have completed all of the preceding self-tests.

The format of this book lends itself to proper pacing. When you're going too slowly, you'll say to yourself, "This stuff is so easy—I'm getting bored." You'll be able to skip a few sections and move on to new material. But when you find yourself pounding your fists against the wall and despairing of ever learning math, that may mean you've been moving ahead a bit too quickly.

If you feel that you don't need to read a particular chapter, you may want to take the self-tests anyway. These provide not only a quick review of the subject matter covered in the chapter, but also a good way of gauging what you already know.

Should you find, on the other hand, that you're having trouble doing a certain type of problem, it will be made clear to you that you need to review an earlier section. For example, no one can do simple division without knowing the multiplication table, so everyone who gets stuck at this point will be sent back to learn that table once and for all. Once that's accomplished, it will be clear sailing through the next few chapters.

This book provides a fast-paced review of arithmetic and elementary algebra, with a smattering of statistics thrown in. It is intended to refresh the memory of the high school or college graduate.

The main emphasis here is on getting you to rely on your own mathematical skills. No longer will you be intimidated trying to calculate tips. No longer will you need to whip out your pocket calculator to do simple arithmetic. And you won't have to wait months to see tangible results. You won't even have to wait weeks. In just a few days your friends and colleagues will notice your new mathematical muscles. So don't delay another minute. Turn to Chapter 2 and just watch those brain cells start to grow.

Acknowledgments

From Steve: Many thanks are due, so I'd like to name names. My longtime editor at Wiley, Judith McCarthy, made hundreds of suggestions to improve and update the book. Authors often hate to change even a word, but Judith's editing has made this second edition a much smoother read. Claire McKean did a thorough copyedit, catching dozens of errors that made it through the first edition, and Benjamin Hamilton supervised the production of the book from copyediting through page proofs.

I owe a large debt of gratitude to my family, especially to my nephews, Jonah and Eric Zimiles. Jonah provided me with a blow-by-blow critique of the strengths and weaknesses of my previous book, *Economics: A Self-Teaching Guide* (Wiley, 1988), on which I was able to build while writing *this* book. And Eric, after having read that book, recognized its format lent itself best to my writing style and encouraged me to write another book. Eric's daughters, Eleni, 11, Justine, 7, and Sophie, 5, have contributed to the new edition by helping me with my math whenever I happened to get stuck.

My father, Jack, a retired math teacher, provided inspiration of another kind. As the oldest living academic perfectionist, he upholds such unattainable standards that one cannot help but feel tolerance for one's own shortcomings and those of just about everyone else. And finally, I wish to thank my sister, Leontine Temsky, for her rationality and common sense in the most uncommon and irrational of times.

From Carolyn: Every opportunity to work with the incredible folks at Wiley has been a pleasure. This project was no exception. My thanks to go Christine O'Connor, Tom Dinse, and Riley Harding for providing the vision for the project, giving me the freedom to make it my own, and guiding me through every step. I'm grateful also to copyeditor Julie Kerr, whose keen eye and infinite patience make the final product so much better.

All the Math You'll Ever Need

1 Getting Started

Far too many Americans are mathematically illiterate. Although many of these people are college graduates, they have trouble doing simple arithmetic. One cannot help but wonder how so many people managed to get so far without having mastered basic arithmetic. Math phobia seems to have become fashionable. People who would never think it amusing to claim not to be able to read or write chuckle as they announce, "I can't do math."

We all have to deal with numbers *sometime*—in banking, on taxes, in choosing a mortgage. Like it or not, numbers are an important part of our lives, and the importance of numerical literacy is increasing in finance, economics, science, government, and more. It is time math stopped intimidating us.

What we'll be doing in this book is going back to basics. We'll focus on the multiplication table. You'll need to memorize it. If you need an even more basic text, you can refer to one such as *Quick Arithmetic: A Self-Teaching Guide, 3rd edition* by Robert A. Carmen and Marilyn J. Carmen (Wiley, 2001).

In *All the Math You'll Ever Need*, the use of complex formulas is generally avoided. Although such formulas have an honored place in mathematics, they rarely need to be memorized. The ones that are used frequently work their way into memory. The others can be looked up when they're needed.

Finally, the use of technical terms is minimized whenever possible. Having the vocabulary to describe mathematical ideas and operations accurately is important to learning but you don't need a lot of fancy language for that. There are no quadratic formulas, logarithmic tables, integrals, or derivatives, and there are only a handful of very simple graphs.

This book was designed to be explored without ever using a calculator or computer. Don't get nervous. You will not be asked to throw away your calculator. Just put it in a safe place for now, to be taken out and used only on proper occasions. A calculator is most effectively used for three tasks: (1) to do calculations that need to be done rapidly, (2) to do repetitive calculations, and (3) to do sophisticated calculations that would take a great deal of time to do without a calculator. Calculators and computers are fast and as accurate as their users allow them to be. Typos are a thing, even on calculators. You need to first know what you want to ask the calculator to do, and then have enough math knowledge to decide if the answer it gives you makes sense.

The trick is to use our calculators for these specific tasks and not for arithmetic functions that we can do in our heads. So put away your calculator and start using your innate mathematical ability.

2 Essential Arithmetic

Every number system (and, yes, there are or have been others) is made up of a set of symbols that we call numbers and one or more operations you can perform with them. Those operations make up what we call *arithmetic*. The basic operation in our number system is addition, the act of putting together. The other operations—multiplication, subtraction, division—are related to, or built from, addition.

1 ADDITION

Addition is, at its heart, about counting. If you have 6 pair of shoes and you buy 3 new pairs, counting will tell you that you now have 9 pairs. You added 6 + 3 and got an answer of 9 by counting. After a while you don't have to count every time, because you get to know that 6 + 3 = 9.

You store a lot of addition facts like that in your memory, but there's a limit to how much memorization can help. You probably know that 4 + 8 = 12, but you're unlikely to memorize the answer to 5,387 + 9,748. Adding larger numbers requires a little more information about our number system.

Place Value

Our number system is a place value system, meaning that the value of a numeral depends on the place it sits in. In the number 444 each 4 has a different meaning. The 4 on the right is in the ones place so it represents 4 ones or simply 4. The 4 on the left is in the hundreds place and represents 4 hundreds or 400. The middle 4 is in the tens place so it represents 4 tens or 40. The number 444 is a shorthand for 400 + 40 + 4.

That expanded form, 400 + 40 + 4, helps to explain how we add large numbers. We add the ones to the ones, the tens to the tens, the hundreds to the hundreds and on up in the place value system. If you need to add 444 + 312, think:

$$\begin{array}{r} 400 + 40 + 4 \\ +\ 300 + 10 + 2 \end{array}$$

Add the 4 ones and the 2 ones to get 6 ones, the 4 tens with 1 ten to get 5 tens and the 4 hundreds with 3 hundreds to get 7 hundreds. Now that would look like this:

$$\begin{array}{r} 400 + 40 + 4 \\ 300 + 10 + 2 \\ \hline 700 + 50 + 6 \end{array}$$

You're probably thinking that you could just write the numbers underneath one another in standard form and add down the columns, and you'd be absolutely correct.

$$\begin{array}{r} 444 \\ +\ 312 \\ \hline 756 \end{array}$$

The reason to think about it in expanded form, at least for a few minutes, comes up when you have to add something like 756 + 968. The basic rule is the same.

$$\begin{array}{r} 7 \text{ hundreds} + 5 \text{ tens} + 6 \text{ ones} \\ 9 \text{ hundreds} + 6 \text{ tens} + 8 \text{ ones} \\ \hline 16 \text{ hundreds} + 11 \text{ tens} + 14 \text{ ones} \end{array}$$

But you can't squeeze 16 (or 11 or 14) into one place. 756 + 968 does not equal 161114. You've got to do some regrouping, or what's commonly called *carrying*. Those 14 ones equal 1 ten and 4 ones. You're going to keep the 4 ones in the ones place and move the ten over to the middle place with the rest of the tens. That will turn

$$\begin{array}{r} 7 \text{ hundreds} + 5 \text{ tens} + 6 \text{ ones} \\ 9 \text{ hundreds} + 6 \text{ tens} + 8 \text{ ones} \\ \hline 16 \text{ hundreds} + 11 \text{ tens} + 14 \text{ ones} \end{array} \quad \text{into} \quad \begin{array}{r} 7 \text{ hundreds} + \begin{matrix} 1 \text{ ten} \\ 5 \text{ tens} \end{matrix} + 6 \text{ ones} \\ 9 \text{ hundreds} + 6 \text{ tens} + 8 \text{ ones} \\ \hline 16 \text{ hundreds} + 12 \text{ tens} + 4 \text{ ones} \end{array}$$

You'll do the same sort of regrouping with the 12 tens. Ten of those tens make 1 hundred, leaving 2 tens in the tens place. You can do this without using the expanded form. Add 6 + 8 to get 14. Put down the 4 and carry the one ten.

$$\begin{array}{r} {\scriptstyle 1} \\ 756 \\ +\ 968 \\ \hline 4 \end{array}$$

Add 1 + 5 + 6 to get 12. Put down the 2 (tens) and carry the 1 (hundred).

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 756 \\ +\ 968 \\ \hline 24 \end{array}$$

Add 1 + 7 + 9 to get 17. The 7 goes in the hundreds place and the 1 (thousand) slides into the thousands place.

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 756 \\ +\ 968 \\ \hline 1{,}724 \end{array}$$

Problem 1:

Add 312 and 423.

Solution:

$$\begin{array}{r} 312 \\ +\ 423 \\ \hline 735 \end{array}$$

All that's necessary is adding the digits in each column: 2 + 3 = 5, 1 + 2 = 3, and 3 + 4 = 7.

Problem 2:

What is the result when 459 is added to 1,276?

Solution:

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 1{,}276 \\ +\ 459 \\ \hline 1{,}735 \end{array}$$

This one requires a little bit of regrouping. Add 6 + 9 to get 15, put down the 5 and carry 1 to the next column. Then 7 + 5 is 12, plus the 1 you carried is 13. Put down the 3 and carry the 1. You can think of the rest as 2 + 4 + 1 = 7 and the 1 thousand comes down unchanged, or you can think of it as 12 + 4 is 16, plus 1 you carried is 17.

Problem 3:

What is the combined total of 9,671 and 2,859?

Solution:

$$\begin{array}{r} {\scriptstyle 1\ \ 1\,1} \\ 9,671 \\ +\,2,859 \\ \hline 12,530 \end{array}$$

Here again you're regrouping. In the ones column, 1 + 9 is 10, so put down the 0 and carry the 1. Then 7 + 5 is 12 plus 1 you carried makes 13. Put down the 3 and carry the 1. Add 6 + 8 + 1 to get 15. Put down the 5 and carry the 1. Finally, 9 + 2 + 1 is 12.

2 MULTIPLICATION

Multiplication is repeated addition. For instance, you probably know 4×3 is 12 because you searched your memory for that multiplication fact. There's nothing wrong with that.

Another way to calculate 4×3 is to think of it as adding four threes, or adding three fours.

$$3 + 3 + 3 + 3 = 12 \qquad \text{or} \qquad 4 + 4 + 4 = 12$$

What about 5×7? Maybe you know it's 35, but you could always do this:

$$7 + 7 + 7 + 7 + 7 = 35 \qquad \text{or} \qquad 5 + 5 + 5 + 5 + 5 + 5 + 5 = 35$$

You do multiplication instead of addition because it's shorter—sometimes much shorter. Suppose you needed to multiply 78×95. If you set this up as an addition problem, you'd have to write 78 copies of 95 before you could even start adding.

Let's set this up as a regular multiplication problem and take a look at the expanded form.

$$\begin{array}{r} 95 \\ \times\ 78 \end{array} \quad \text{becomes} \quad \begin{array}{r} 90 + 5 \\ \times\ 70 + 8 \end{array}$$

The key to this multiplication is you have to multiply 8×5 and 8×90 and then multiply 70×5 and 70×90, and add up all the results. Don't get discouraged, because there is a condensed form.

The first set of numbers we'd multiply would be 8 × 5. You probably know, or can figure out, that's 40. (We'll focus on all the multiplication facts you should memorize in Chapter 3, "Focus on Multiplication.") Then we'd multiply 8 × 90, which just means multiplying 8 × 9 and putting a zero at the end. Whenever you multiply a number that ends in zero, you can deal with the non-zero parts and add the zero at the end. (See Chapter 5, "Mental Math" for more on that shortcut.) 8 × 9 =72 so 8 × 90 = 720. Next would come 70 × 5. 7 × 5 = 35 so 70 × 5 =350. The last multiplication would be 70 × 90. Multiply 7 × 9 = 63, and then add a zero for the 70 and another zero for the 90. 70 × 90 = 6,300. Add up 6,300 + 350 + 720 + 40 to get 7,410.

$$\begin{array}{r} 95 \\ \times\ 78 \\ \hline 40 \\ 720 \\ 350 \\ 6300 \\ \hline 7410 \end{array}$$

Here's how to write it more compactly. Multiply 8 × 5 = 40, put down the 0 and carry the 4. 8 × 9 = 72 and the 4 we carried makes 76. Write the 76 in front of that 0 you put down and you see 760. This 760 is the 40 and the 720 combined. Now, you need to multiply 95 by 70, which means multiply by 7 and add a zero. So put the zero down first, under the 0 of the 760. Then 7 × 5 = 35. Put down the 5 to the left of the 0 and carry the 3. 7 × 9 = 63 plus the 3 you carried is 66. Write the 66 in front of the 50 and you've got 6,650, which is the 350 and 6300 combined. Add the two lines, and you're done.

$$\begin{array}{rrrr} {\scriptstyle 4} & & {\scriptstyle 3} & {\scriptstyle 3} \\ 95 & 95 & 95 & 95 \\ \times 78 & \times 78 & \times 78 & \times 78 \\ \hline 760 & 760 & 760 & 760 \\ & 0 & 50 & 6650 \\ & & & \hline & & & 7410 \end{array}$$

As you can see, a long multiplication problem can be broken down into a series of simple multiplication problems. It's important to have basic multiplication facts in memory, so you don't have to spend time doing the repeated addition every time. You'll learn more about that in the next chapter.

Problem 1:

Multiply 73 by 5.

Solution:

$$\begin{array}{r} {\scriptstyle 1} \\ 73 \\ \times\, 5 \\ \hline 365 \end{array}$$

Begin with $5 \times 3 = 15$. Put down the 5 and carry 1. Then 5×7 is 35 plus the 1 you carried is 36.

Problem 2:

Find the product of 86 and 12.

Solution:

First multiply 86 by 2.

$$\begin{array}{r} {\scriptstyle 1} \\ 86 \\ \times 12 \\ \hline 172 \end{array}$$

Multiplying 2×6 gives you 12, so put down the 2 and carry 1. Then 2×8 is 16 plus 1 you carried is 17.

Place a zero at the end of the second line, or if you prefer, just move one space right, and multiply 1×86, which obviously is 86. Add the columns to complete the job.

$$\begin{array}{r} 86 \\ \times 12 \\ \hline {\scriptstyle 1} \\ 172 \\ 860 \\ \hline 1{,}032 \end{array}$$

Problem 3:

What is the result when 125 is multiplied by 32?

Solution:

Multiplying 125 by 2 requires a little bit of carrying.

$$\begin{array}{r} {\scriptstyle 1} \\ 125 \\ \times\ 32 \\ \hline 250 \end{array}$$

Place a zero on the second line, or move one space right, then multiply 3×125. Add down each column, regrouping where necessary.

$$\begin{array}{r} {\scriptstyle 1} \\ 125 \\ \times 32 \\ \hline {\scriptstyle 1} \\ 250 \\ {\scriptstyle 1} \\ 3750 \\ \hline 4{,}000 \end{array}$$

Ready to test yourself? Try Self-Test 2.1.

SELF-TEST 2.1

1. Add 453 and 975.
2. Find the sum of 1,864 and 798.
3. Multiply 561 by 92.
4. What is the product of 891 and 30?
5. Multiply 135×112.

If the first two gave you trouble, review Frame 1. If you got any of the last three wrong, review Frame 2. If you've got this, move on!

3 SUBTRACTION

Subtraction is the inverse, or opposite, of addition. Addition puts together. Subtraction takes apart. If you buy a carton of 12 eggs and you use 4 of them to make breakfast, how many eggs are left? $12 - 4 = 8$ if you count the remaining eggs.